P131

[1] $yz \perp x\text{-axis}$

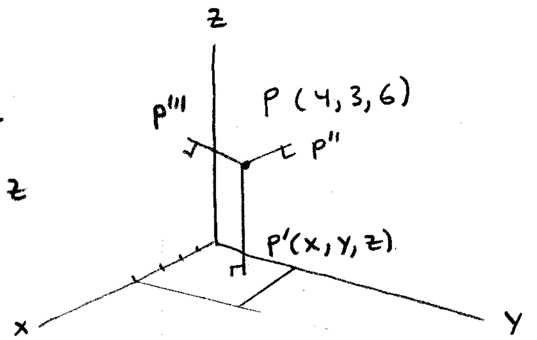
$y\text{-axis} \perp xz$

P132

[2] $P'(4,3,0)$ $PP' \perp xy$

$P''(4,0,6)$ $PP'' \perp yz$

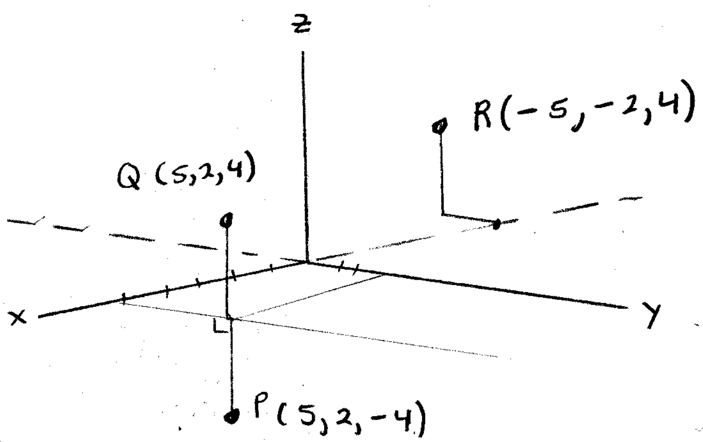
$P'''(0,3,6)$ $PP''' \perp xz$



[3]

$Q(5,2,4)$

$R(-5,-2,4)$



[4.1] $(2-3,\ 3+1,\ 4-2) = (-1,4,2)$

[4.2] $(-2-3,\ -3+1,\ -4-2) = (-5,-2,-6)$

[4.3] $(-3-3,\ 1+1,\ -2-2) = (-6,2,-4)$

P133

[5] $4-\alpha = -1$ $\Rightarrow$ $\alpha = 5$

$2-\beta = 3$ $\beta = -1$

$-3-\gamma = 5$ $\gamma = -8$

$\therefore (1,1,1) \longrightarrow (-4,2,9)$

[6] Intentionally left out

P134

[1.1] $d^2 = (3-2)^2 + (-2-0)^2 + (4-3)^2$

$= 1 + 4 + 1$

$= 6$

$\therefore d = \sqrt{6}$

[1.2] $d^2 = (0+4)^2 + (2-0)^2 + (3-1)^2$

$= 16 + 4 + 4$

$= 24$

$d = 2\sqrt{6}$

P 135

[2.1] $(x-2)^2 + (y-0)^2 + (z+5)^2 = 36$

[2.2] To find the radius, note that the distance to yz-plane is 4. Then

$(x-4)^2 + (y+3)^2 + (z-5)^2 = 16$

P 136

[3] $x^2 + y^2 + z^2 - 4x + 6y + 2z = 11$

$(x^2 - 4x + 4) + (y^2 + 6y + 9) + (z^2 + 2z + 1) = 11 + 4 + 9 + 1$

$(x-2)^2 + (y+3)^2 + (z+1)^2 = 5^2$

$\therefore$ $C(2, -3, -1)$, $r = 5$.

[4] $x^2 + y^2 + z^2 = 16$

Interior $\{(x,y,z) \ni x^2 + y^2 + z^2 < 16\}$

Exterior $\{(x,y,z) \ni x^2 + y^2 + z^2 > 16\}$

[5] $C(2,1,-3)$, $r = 1$

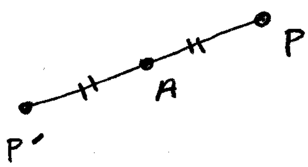
$C(2,1,-3)$, $r = 4$

$R = \{(x,y,z) \ni 1 < (x-2)^2 + (y-1)^2 + (z+3)^2 < 16\}$

P 136 Exercises

[1] P' sym to $P(5,-2,6)$ w.r.t. $A(3,2,-4)$

A must be midpoint of $P'P$.



$$x = \frac{5+3}{2},\ y = \frac{-2+2}{2},\ z = \frac{6-4}{2}$$

$\therefore$ $x = 4,\ y = 0,\ z = 1,\quad P'(4,0,1)$

[2] Get Pts on x- and y-axes that are equidistant from $A(4,5,3)$ and $B(3,-2,5)$.

P on x-axis $\equiv$ $P(x,0,0)$

Q on y-axis $\equiv$ $Q(0,y,0)$

x-coord

$$PA^2 = (x-4)^2 + (0-5)^2 + (0-3)^2 = x^2 - 8x + 16 + 29 + 9$$

$$PB^2 = (x-3)^2 + (0+2)^2 + (0-5)^2 = x^2 - 6x + 9 + 4 + 25$$

$$PA^2 = PB^2 \equiv x^2 - 8x + 54 = x^2 - 6x + 38$$

$$\equiv 2x - 16 = 0$$

$$\equiv \boxed{x = 8}$$

Y-COORD

$$QA^2 = (4-0)^2 + (5-y)^2 + (3-0) \equiv 16 + 25 - 10y + y^2 + 9$$

$$QB^2 = (3-0)^2 + (-2-y)^2 + (5-0)^2 \equiv 9 + 4 + 4y + y^2 + 25$$

$$QA^2 = QB^2 \equiv y^2 - 10y + 50 = y^2 + 4y + 38$$

$$\equiv 16y - 12 = 0$$

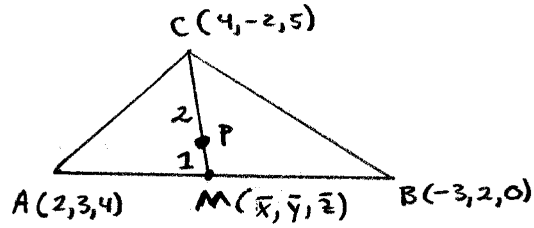
$$\equiv y = \frac{12}{16}$$

$$\equiv \boxed{y = \frac{3}{4}}$$

$\therefore$ $P(8,0,0)$ and $Q(0,\frac{3}{4},0)$

P136, ctd

[3.1] Let $P(x, y, z)$ be the centroid of ΔABC.



Recall that the centroid is $\frac{2}{3}$ length of median from vertex.

Coords of M

$$\bar{x} = \frac{2-3}{2}, \quad \bar{y} = \frac{3+2}{2}, \quad \bar{z} = \frac{4+0}{2}$$

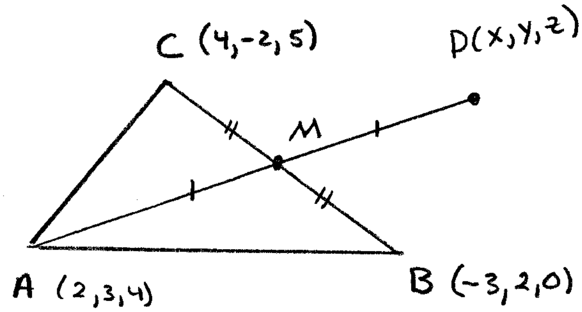
$$\boxed{M\left(-\tfrac{1}{2}, \tfrac{5}{2}, 2\right)}$$

Now $\frac{CP}{PM} = \frac{2}{3}$, so that

$$x = \frac{2(-\frac{1}{2}) + 4}{3}, \quad y = \frac{2(\frac{5}{2}) - 2}{3}, \quad z = \frac{2(2) + 5}{3}$$

$\therefore$ $P(1, 1, 3)$

[3.2]



M: $x = \frac{4-3}{2}$, $y = \frac{-2+2}{2}$, $z = \frac{5+0}{2}$

$$\boxed{M\left(\tfrac{1}{2}, 0, \tfrac{5}{2}\right)}$$

Let M be midPT AD,

$$\frac{1}{2} = \frac{x+2}{2}, \quad 0 = \frac{y+3}{2}, \quad \frac{5}{2} = \frac{4+z}{2}$$

$$x = -1, \quad y = -3, \quad z = 1$$

So

$\therefore$ $D(-1, -3, 1)$

pps 136-

p 137

[6.1]

center at (a, b, c)

$$a = \frac{-2+4}{2} = 1$$

$$b = \frac{1-3}{2} = -1$$

$$c = \frac{5-1}{2} = 2$$

$\boxed{C\ (1, -1, 2)}$

$$r^2 = (1-4)^2 + (-1+3)^2 + (2+1)^2$$

$$= 9 + 4 + 9$$

$$\boxed{r^2 = 22}$$

$$\therefore \quad (x-1)^2 + (y+1)^2 + (z-2)^2 = 22$$

[6.2] Since $P(-5,1,4)$ on sphere, coordinates of center C are $(-a,b,c)$ where a,b,c all positive.

{ this says that center of sphere must lie on same Side of yz-plane as the point $P(-5,1,4)$. }

$r = a = b = c$ = distance from Center to $P(-5,1,4)$.

$(-a+5)^2+(b-1)^2+(c-4)^2 = r^2$, distance formula

$(-r+5)^2+(r-1)^2+(r-4)^2 = r^2$, substitute r for a,b,c

$r^2+r^2+r^2-10r-2r-8r+42 = r^2$

$2r^2-20r+42=0$

$r^2-10r+21=0$

$(r-3)(r-7)=0$

$r=3,\ r=7,$

So center is $C(-3,3,3)$ or $C(-7,7,7)$.

$\therefore \quad (x+3)^2+(y-3)^2+(z-3)^2 = 9$

and $\quad (x+7)^2+(y-7)^2+(z-7)^2 = 49$

are both solutions.

[7.1] $A(-1,0,0)$

$B(2,0,0)$

$$\frac{AP}{PB} = \frac{1}{2}.$$

Let $P(x,y,z)$ be a point on the figure. Then,

$$AP^2 = (x+1)^2 + y^2 + z^2$$

$$PB^2 = (x-2)^2 + y^2 + z^2$$

$$AP^2 = \frac{1}{4} PB^2.$$

$$x^2 + 2x + 1 + y^2 + z^2 = \frac{1}{4}(x^2 - 4x + 4 + y^2 + z^2)$$

$$4x^2 + 8x + 4 + 4y^2 + 4z^2 = x^2 - 4x + 4 + y^2 + z^2$$

$$3x^2 + 12x + 3y^2 + 3z^2 = 0$$

$$x^2 + 4x + y^2 + z^2 = 0$$

$$(x^2 + 4x + 4) + y^2 + z^2 = 4$$

$$\therefore \quad (x+2)^2 + (y-0)^2 + (z-0)^2 = 4$$

I.e. Figure is a circle radius 2

center at $(-2,0,0)$.

[7.2] $A(1,2,0)$, $B(-1,4,2)$

$AP^2 + BP^2 = 38$

<u>soln</u>

Let $P(x,y,z)$ be point on figure.

$$AP^2 = (x-1)^2 + (y-2)^2 + z^2$$

$$BP^2 = (x+1)^2 + (y-4)^2 + z^2$$

$$AP^2 + BP^2 = 38$$

$$x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 + x^2 + 2x + 1 + y^2 - 8y + 16 + z^2 = 38$$

$$2x^2 + 2y^2 - 12y + 2z^2 + 22 = 38$$

$$x^2 + y^2 - 6y + z^2 = 8$$

$$(x-0)^2 + (y-3)^2 + (z-0)^2 = 8 + 9$$

$$\therefore \quad x^2 + (y-3)^2 + z^2 = 17$$

I.e. circle radius $\sqrt{17}$ center $(0,3,0)$.